Harnessing non-Markovian quantum memory by environmental coupling

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Controlling the non-Markovian dynamics of open quantum systems is essential in quantum information technology since it plays a crucial role in preserving quantum memory. Albeit in many realistic scenarios the quantum system can simultaneously interact with composite environments, this condition remains little understood, particularly regarding the effect of the coupling between environmental parts. We analyze the non-Markovian behavior of a qubit interacting at the same time with two coupled single-mode cavities which in turn dissipate into memoryless or memory-keeping reservoirs. We show that increasing the control parameter, that is the two-mode coupling, allows for triggering and enhancing a non-Markovian dynamics for the qubit starting from a Markovian one in absence of coupling. Surprisingly, if the qubit dynamics is non-Markovian for zero control parameter, increasing the latter enables multiple transitions from non-Markovian to Markovian regimes. These results hold independently on the nature of the reservoirs. This work highlights that suitably engineering the coupling between parts of a compound environment can efficiently harness the quantum memory, stored in a qubit, based on non-Markovianity.

I. INTRODUCTION

A thorough understanding of the dynamics of an open quantum system has experienced a long term pursuit [1] and nowadays attracts ever-increasing attention due to the development of quantum information technology [2, 3] that employs open quantum systems as basic resource. In the theory of open quantum systems, the non-Markovian dynamics is one of main concerns being linked to the preservation of quantum memory stored in a quantum system [1, 4, 5]. It arises in many realistic situations [4–23], and also proves useful in quantum information processing such as quantum state engineering, quantum channel capacity and quantum control [4, 24–28]. The degree of a non-Markovian evolution, the so-called non-Markovianity, can be quantified by different measures based on dynamical features of the system capable to grasp the memory effects of the environment on the system evolution [28–35]. So far, many factors that can trigger and modify the non-Markovian dynamics have been found, as strong system-environment coupling, structured reservoirs, low temperatures, and initial system-environment correlations [1, 4–6, 36–40]. Apart from these mechanisms, some other peculiar conditions such as classical environments [41, 42] and environmental initial correlations [43, 44] have also been predicted and experimentally demonstrated [42, 44] enabling emergence of non-Markovianity.

In the conventional study, one usually considers the quantum system being coupled to a single environment [1, 4, 5]. However, in several realistic scenarios the system may be simultaneously influenced by many environments [45–49]. For instance, in a quantum dot the electron spin may be affected strongly by the surrounding nuclei and weakly by the phonons [45]. The neighbor nitrogen impurities constitute the principal bath for a nitrogen-vacancy center, while the carbon-13 nuclear spins also have some interaction with it [46]. A similar situation also occurs for a single-donor electron spin in silicon [47]. Motivated by these practical situations [45–47], some efforts have been devoted to study the effects of multiple environments on the dynamics of an open system [48–52]. Quantum interference effects have been found to occur between independent reservoirs when all of them interact with a quantum system and are in non-Markovian regimes, which qualitatively modify the dynamics of the interested system [48]. The dynamics of a spin simultaneously coupled to two decoherence channels, one Markovian and the other non-Markovian, has been analyzed with respect to the different decoherence mechanisms [49]. As is known, a qubit (i.e., a two-level system) interacting with a single vacuum bosonic reservoir may exhibit Markovian or non-Markovian dynamics depending on the strength of the system-reservoir coupling [1]. By contrast, if the qubit simultaneously interacts with several reservoirs, its dynamics can be always non-Markovian provided that the number of the contributing reservoirs is greater than a critical value [50]. The dynamics of a qubit coupled to a hierarchical environment made of a single-mode cavity and a structured reservoir with Lorentzian spectral density has been studied, showing that a shorter (longer) memory time of the reservoir does not generally mean a smaller (larger) non-Markovianity of the system [51].

A new analytical method based on a phase space representation of the system density matrix has been also proposed to study the dynamics of a discrete system in a two-tiered non-Markovian environment [52].

In the treatments of composite environments [48–50], the role of the coupling between environmental parts is not typically taken into account. Despite this, the knowledge of how this environmental parameter influences the non-Markovian character of an open quantum system would provide insightful developments for engineering and controlling quantum memories for possible technological applications. Therefore, this aspect deserves careful investigation, possibly starting from a paradigmatic model where it can simply emerge and be understood. Here we choose a model which complies with this requirement, namely a qubit interacting at the same time with two coupled single-mode cavities which in turn dissipate photons into their own memoryless (Markovian) or memory-keeping (non-Markovian) reservoirs. This system finds its natural implementation in nowadays technologies of circuit quantum electrodynamics [53] and also in simulating all-optical setups [54]. We shall show that the coupling strength between the two modes can harness the qubit non-Markovianity in different and even counter-intuitive ways, independently of the nature of the reservoirs. The paper is organized as follows. In Sec. II we describe the system of interest. In Sec. III we study the case of memoryless reservoirs, while in Sec. IV we investigate the case of memory-keeping reservoirs. In Sec. V we summarize our conclusions.

II. THE SYSTEM

Our global system consists of a qubit s simultaneously interacting with two environments $\mathcal{E}_1$ and $\mathcal{E}_2$. To fix the ideas, we model each environment $\mathcal{E}_n$ ($n = 1, 2$) as a bosonic mode m_n decaying to a zero-temperature bosonic reservoir $\mathcal{R}_n$, as depicted in Fig. 1. The qubit is here meant as the quantum memory whose efficiency is to be quantified by its non-Markovianity, that is by the degree of non-Markovian evolution. The interaction of the two environments is due to the coupling of the two bosonic cavity modes, which instead plays the role of a control parameter for the non-Markovianity of the qubit. The coupling strength of the qubit with each mode m_n is κ_n, while Ω denotes the coupling between the two modes. For the sake of simplicity, we assume that the two modes have the same frequency ω_c which in turn is equal to the qubit transition frequency ω_0, that is $\omega_0 = \omega_c$.

The total Hamiltonian is given by ($\hbar = 1$)

$$\hat{H} = \hat{H}_s + \sum_{n=1}^{2}[\hat{H}_{m_n} + \hat{H}_{\mathcal{R}_n} + \hat{H}_{sm_n} + \hat{H}_{m_n\mathcal{R}_n}]$$
$$+ \hat{H}_{m_1m_2}, \tag{1}$$

where $\hat{H}_s = (\omega_0/2)\hat{\sigma}_z$ is the qubit Hamiltonian, $\hat{H}_{m_n} = \omega_c \hat{a}_n^\dagger \hat{a}_n$ the mode Hamiltonian, $\hat{H}_{\mathcal{R}_n} = \sum_k \omega_{n,k} \hat{b}_{n,k}^\dagger \hat{b}_{n,k}$
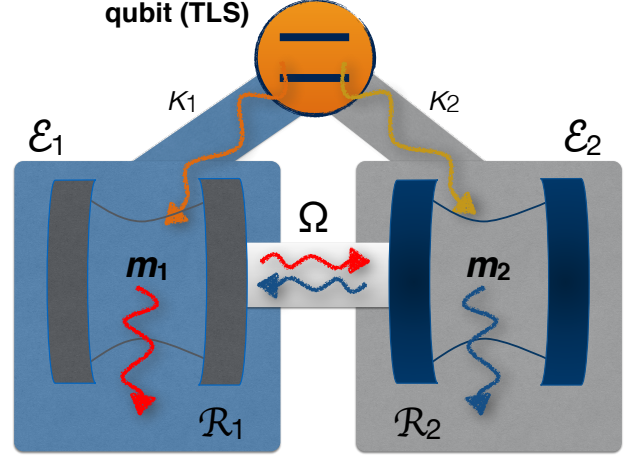

FIG. 1: **Pictorial description of the global system.** A qubit, i.e. a two-level system (TLS), simultaneously interacts with two environments $\mathcal{E}_n$ ($n = 1, 2$), each containing a single cavity mode m_n that in turns is affected by a vacuum (zero temperature) reservoir $\mathcal{R}_n$. The qubit is directly coupled to each mode m_n with strength κ_n. The two cavity modes m_1, m_2 are coupled with strength Ω.

the reservoir Hamiltonian, $\hat{H}_{sm_n} = \kappa_n(\hat{a}_n^\dagger \hat{\sigma}_- + \hat{a}_n \hat{\sigma}_+)$ the qubit-mode interaction Hamiltonian, $\hat{H}_{m_n\mathcal{R}_n} = \sum_k g_{n,k}(\hat{a}_n \hat{b}_{n,k}^\dagger + \hat{a}_n^\dagger \hat{b}_{n,k})$ the mode-reservoir interaction Hamiltonian and $\hat{H}_{m_1m_2} = \Omega(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger)$ the interaction Hamiltonian between the two modes. In the expressions above $\hat{\sigma}_z = |1\rangle\langle 1| - |0\rangle\langle 0|$ is a Pauli operator for the system with transition frequency ω_0, $\hat{\sigma}_\pm$ represent the raising and lowering operators of the qubit, $\hat{a}_n$ ($\hat{a}_n^\dagger$) the annihilation (creation) operator of mode m_n. Furthermore, in the Hamiltonians involving the reservoirs $\hat{b}_{n,k}$ ($\hat{b}_{n,k}^\dagger$) is the annihilation (creation) operator of field mode k with frequency $\omega_{n,k}$ of reservoir $\mathcal{R}_n$, and $g_{n,k}$ denotes the coupling of the mode m_n with the mode k of its own reservoir $\mathcal{R}_n$. In the interaction picture, the total Hamiltonian can be expressed as

$$\hat{H}_{\text{int}} = \sum_{n=1}^{2} \hat{H}_{sm_n} + \hat{H}_{m_1m_2}$$
$$+ \sum_{n=1}^{2} \sum_k g_{n,k}(\hat{a}_n \hat{b}_{n,k}^\dagger e^{i\Delta_{n,k}t} + \hat{a}_n^\dagger \hat{b}_{n,k} e^{-i\Delta_{n,k}t}) \tag{2}$$

where $\Delta_{n,k} = \omega_{n,k} - \omega_0$.

The reservoirs $\mathcal{R}_n$ of the global system can be either memoryless (Markovian) or memory-keeping (non-Markovian). Depending on the kind of reservoir, different methods are used to obtain the reduced dynamics of the qubit. In the following two sections we study these two cases.

III.   MEMORYLESS RESERVOIRS

In this section we consider both the reservoirs $\mathcal{R}_n$ as vacuum Markovian ones, their correlation times being much smaller than the single-mode relaxation times. Although our system can be exactly solved (see Sec. IV), we first treat it under the Markov approximation since this analysis constitutes a strategical first step in order to strongly evidence the crucial role of the two-mode coupling parameter to harness quantum non-Markovianity for the dynamics of the qubit even under this condition. In this case, the density operator $\rho(t)$ of the qubit plus the two modes obeys the following master equation [1]

$$
\begin{aligned}
\dot{\rho}(t) = {}& -i[\hat{H}, \rho(t)] \\
& - \sum_{n=1}^{2} \frac{\Gamma_n}{2}[a_n^\dagger a_n \rho(t) - 2a_n \rho(t) a_n^\dagger + \rho(t) a_n^\dagger a_n], \quad (3)
\end{aligned}
$$

where $\dot{\rho}(t) \equiv d\rho(t)/dt$, $\hat{H}$ is given by Eq. (1) without the terms involving the reservoirs and Γ_n denotes the decay rate of the mode m_n. We initially take the qubit in its excited state $|1\rangle_s$ and both modes in the ground states $|00\rangle_{m_1,m_2}$, so that the initial overall state is $\rho(0) = |100\rangle\langle100|$, where the first, second and third element correspond to the qubit s, mode m_1 and mode m_2, respectively. Since there exist at most one excitation in the total system at any time, we can make the ansatz for $\rho(t)$ in the form

$$
\rho(t) = (1 - \lambda(t))|\psi(t)\rangle\langle\psi(t)| + \lambda(t)|000\rangle\langle000|, \quad (4)
$$

where $0 \leq \lambda(t) \leq 1$ with $\lambda(0) = 0$ and $|\psi(t)\rangle = h(t)|100\rangle + c_1(t)|010\rangle + c_2(t)|001\rangle$ with $h(0) = 1$ and $c_1(0) = c_2(0) = 0$. It is convenient to introduce the unnormalized state vector [55]

$$
\begin{aligned}
\left|\widetilde{\psi}(t)\right\rangle &\equiv \sqrt{1 - \lambda(t)}\,|\psi(t)\rangle \\
&= \widetilde{h}(t)|100\rangle + \widetilde{c}_1(t)|010\rangle + \widetilde{c}_2(t)|001\rangle, \quad (5)
\end{aligned}
$$

where $\widetilde{h}(t) \equiv \sqrt{1 - \lambda(t)}\,h(t)$ represents the probability amplitude of the qubit and $\widetilde{c}_n(t) \equiv \sqrt{1 - \lambda(t)}\,c_n(t)$ that of the mode m_n being in their excited states. In terms of the unnormalized state vector we then get

$$
\rho(t) = |\widetilde{\psi}(t)\rangle\langle\widetilde{\psi}(t)| + \lambda(t)|000\rangle\langle000|. \quad (6)
$$

Inserting this expression in Eq. (3), the time-dependent amplitudes $\widetilde{h}(t)$, $\widetilde{c}_1(t)$, $\widetilde{c}_2(t)$ of Eq. (5) are determined by a set of differential equations as

$$
\begin{aligned}
i\frac{d\widetilde{h}(t)}{dt} &= \omega_0 \widetilde{h}(t) + \kappa_1 \widetilde{c}_1(t) + \kappa_2 \widetilde{c}_2(t), \\
i\frac{d\widetilde{c}_1(t)}{dt} &= \left(\omega_c - \frac{i}{2}\Gamma_1\right)\widetilde{c}_1(t) + \kappa_1 \widetilde{h}(t) + \Omega \widetilde{c}_2(t), \\
i\frac{d\widetilde{c}_2(t)}{dt} &= \left(\omega_c - \frac{i}{2}\Gamma_2\right)\widetilde{c}_2(t) + \kappa_2 \widetilde{h}(t) + \Omega \widetilde{c}_1(t). \quad (7)
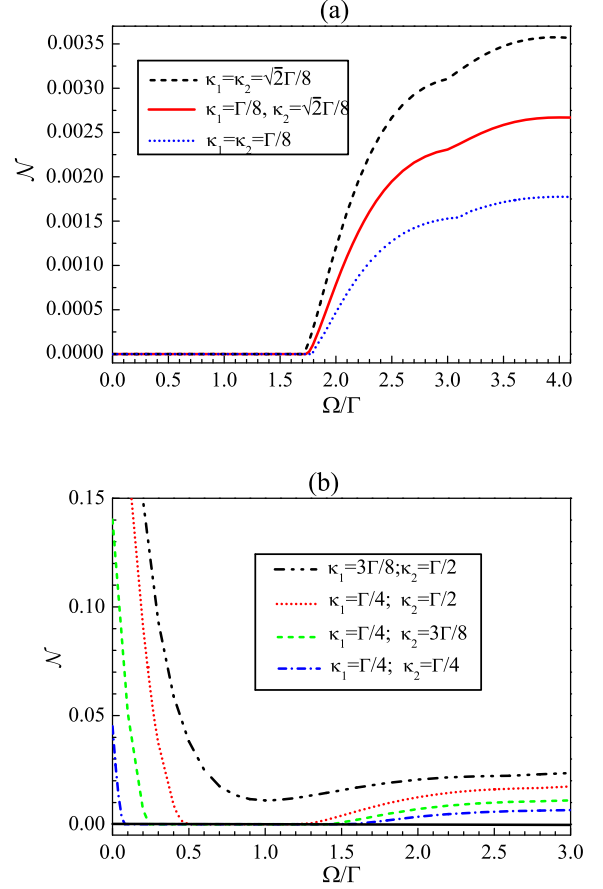\end{aligned}
$$

FIG. 2: Non-Markovianity measure $\mathcal{N}$ as a function of the coupling constant Ω/Γ between the two modes for (a) weak and (b) strong system-mode coupling regimes.

The above differential equations can be solved by means of standard Laplace transformations combined with numerical simulations to obtain the reduced density operators of the qubit as well as of each of the modes.

To quantify the non-Markovianity we adopt a measure based on the dynamics of the trace distance between two different initial states $\rho_1(0)$ and $\rho_2(0)$ of an open system [29]. A Markovian evolution can never increase the distinguishability between different initial states in terms of their trace distance, hence a nonmonotonic time behavior of the latter would signify non-Markovian dynamics of the system. Such a measure is consistent with the interpretation of non-Markovianity in terms of a backflow of information from the environment to the system, which is responsible for the distance (state-distinguishability) growth. Based on this concept, the non-Markovianity can be quantified by a measure $\mathcal{N}$ defined as [29]

$$
\mathcal{N} = \max_{\rho_1(0),\rho_2(0)} \int_{\sigma>0} \sigma[t, \rho_1(0), \rho_2(0)]dt, \quad (8)
$$

in which $\sigma[t, \rho_1(0), \rho_2(0)] = dD[\rho_1(t), \rho_2(t)]/dt$ is the rate

of change of the trace distance given by

$$D[\rho_1(t), \rho_2(t)] = \frac{1}{2}\text{Tr}|\rho_1(t) - \rho_2(t)|, \qquad (9)$$

where $|A| = \sqrt{A^\dagger A}$. In order to evaluate $\mathcal{N}$, we have to find a specific pair of optimal initial states to maximize the time derivative of the trace distance. In Ref. [56], it is proved that the pair of optimal states is associated with two antipodal pure states on the surface of the Bloch sphere. We thus adopt a pair of initial states $\rho_{1,2}(0) = |\psi_{1,2}(0)\rangle\langle\psi_{1,2}(0)|$ with $|\psi_{1,2}(0)\rangle = (|0\rangle \pm |1\rangle)\sqrt{2}$ as the optimal ones throughout the paper. This allows us to obtain the time derivative of the trace distance in the simple form $\sigma[t, \rho_1(0), \rho_2(0)] = d|\widetilde{h}(t)|/dt$.

In the absence of the coupling between m_1 and m_2, the qubit exhibits Markovian dynamics when the couplings of the qubit with the two modes in terms of κ_1/Γ_1 and κ_2/Γ_2 are weak. In this case, we show that the introduction of mode-mode coupling with sufficient strength Ω can transform the Markovian dynamics to the non-Markovian one. In Fig. 2(a), we plot the non-Markovianity $\mathcal{N}$ as a function of the scaled coupling strength Ω/Γ between the two modes for different values of κ_1 and κ_2 ($\Gamma_1=\Gamma_2=\Gamma$ is assumed along the paper). As shown in the figure, the system exhibits Markovian dynamics, individuated by $\mathcal{N} = 0$, until the two modes are weakly coupled and below a certain threshold. However, when Ω/Γ exceeds this threshold the Markovian dynamics of the qubit changes to non-Markovian one (i.e., $\mathcal{N} > 0$). In general, non-Markovianity increases with Ω/Γ for the given values of κ_1 and κ_2 and is also proportional to κ_1, κ_2 for a fixed Ω/Γ. Therefore, the coupling of the two modes can trigger the non-Markovian dynamics of the system.

On the other hand, if the qubit-mode couplings κ_1, κ_2 are strong the qubit exhibits non-Markovian dynamics without the need of mode-mode coupling. Under these conditions, how the additional coupling of the two modes influences the system non-Markovianity is to be revealed. From the above discussion about the case of weak qubit-mode couplings, one might expect that the mode-mode coupling would enhance the non-Markovianity of the system. However, as shown in Fig. 2(b), for different κ_1, κ_2 the relation between the non-Markovianity $\mathcal{N}$ and the mode-mode coupling strength is nonmonotonic. Increasing Ω/Γ from zero, the non-Markovianity first diminishes to a minimal value and then rises. Remarkably, for some smaller values of κ_1 and κ_2, the non-Markovianity can even decrease to zero (Markovian regime), remain zero for a finite range of Ω/Γ and then recover nonzero values with a further increase of Ω/Γ. The mode-mode coupling is thus able not only to enhance the memory effects of the overall environment but also to restrain them.

For a comprehensive understanding of the effects of system-mode and mode-mode couplings on the qubit dynamics, in Fig. 3 we display the phase diagram in the κ-Ω plane ($\kappa_1 = \kappa_2 = \kappa$) of the transitions between Markovian and non-Markovian dynamics. In the strong qubit-
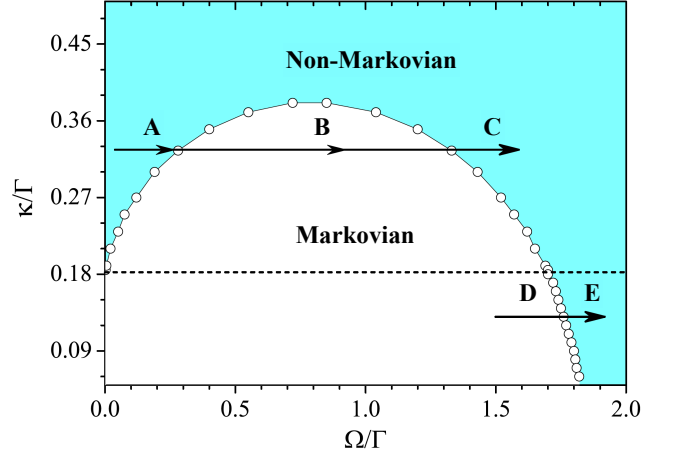

FIG. 3: Phase diagram in the κ/Γ-Ω/Γ plane for the crossover between Markovian and non-Markovian dynamics. The colored regions represent the non-Markovian dynamics, while the white regions denote the Markovian dynamics. The dotted line near $\kappa/\Gamma = 0.18$ divides the weak and strong couplings between the system and the modes and above (below) which is the strong (weak) system-modes coupling regime.

mode coupling regime (above the dotted line), the qubit may experience two transitions: from non-Markovian to Markovian and again to non-Markovian dynamics (e.g., from point A to B and then to C) with an increase of Ω. In the weak qubit-mode coupling regime (below the dotted line), an increase of the mode-mode coupling can drive the Markovian dynamics to the non-Markovian one (e.g., from point D to E). Moreover, the smaller the κ, the larger the Ω required to activate the non-Markovian dynamics.

So far, we have shown that the mode-mode coupling can trigger and modify the non-Markovianity of the system. In fact, the two modes can be regarded as effective memories of the overall environment since the compensated rate $W(t)$ of their population changes, given by [57]

$$W(t) \equiv \frac{d\left(\sum_{n=1}^{2}|\widetilde{c}_n(t)|^2\right)}{dt} + \sum_{n=1}^{2}\Gamma_n|\widetilde{c}_n(t)|^2, \qquad (10)$$

completely determines the qubit non-Markovian dynamics. The meaning of Eq. (10) can be explained as follows. The energy dissipations are one-way from the modes to their memoryless reservoirs so that Γ_1, Γ_2 are always positive. If the energy of the two modes decreases (i.e., $d\left(\sum_{n=1}^{2}|\widetilde{c}_n(t)|^2\right)/dt < 0$) and this decrease is not compensated by the dissipation of the modes, quantified by $\sum_{n=1}^{2}\Gamma_n|\widetilde{c}_n(t)|^2$, then $W(t) < 0$. This situation can only happen when part of the two-mode energy has come back to the quantum system, thus a negative value of $W(t)$ identifies a back-action (or back-flow of information) and
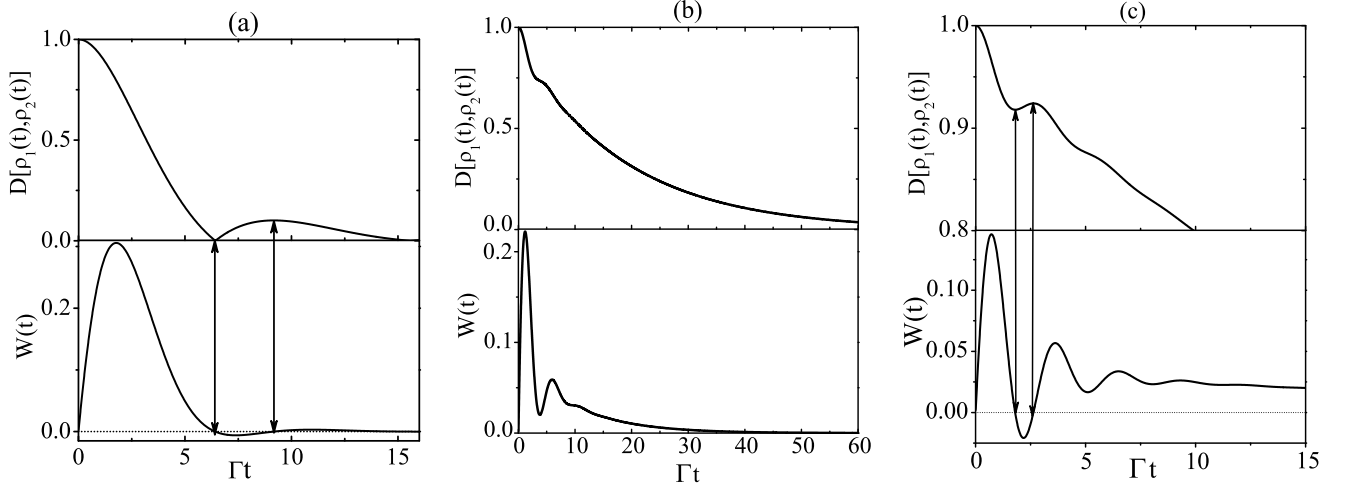
FIG. 4: Dynamics of the trace distance $D[\rho_1(t),\rho_2(t)]$ (top plane) and the witness $W(t)$ (unit s^{-1}) of Eq.(10) (bottom plane), for $\kappa_1 = \kappa_2 = 0.3\Gamma$ and $\Omega = 0$ (a), $\Omega = \Gamma$ (b) and $\Omega = 2\Gamma$ (c).

therefore onset of non-Markovianity. Therefore, the results discussed above indicate that the coupling between the two environmental modes, the memories, can enhance or inhibit their own memory effects on the qubit. As a further verification, in Fig. 4 we compare the dynamics of the trace distance $D[\rho_1(t),\rho_2(t)]$ of Eq. (9) and of the rate $W(t)$ to directly demonstrate that the negativity of $W(t)$ assesses qubit non-Markovianity. To this aim, we choose three points in the κ-Ω phase diagram (Fig. 3) with the same $\kappa = 0.3\Gamma$, while $\Omega = 0$, Γ and 2Γ, respectively. These points pass through the regime transitions non-Markovian $\to$ Markovian $\to$ non-Markovian as Ω increases. As expected, $D[\rho_1(t),\rho_2(t)]$ exhibit oscillations for $\Omega = 0$ and $\Omega = 2\Gamma$ (see Fig. 4, panels (a) and (c)), while asymptotically decay to zero for $\Omega = \Gamma$ (Fig. 4(b)). The witness $W(t)$ becomes negative at the points where $D[\rho_1(t),\rho_2(t)]$ begins to grow and remains negative during the whole time interval when $D[\rho_1(t),\rho_2(t)]$ increases, which entails an information backflow from the modes to the qubit. Differently, $W(t)$ remains positive when $D[\rho_1(t),\rho_2(t)]$ asymptotically decays, as in Fig. 4(b). It is worth to notice that, although the qubit undergoes non-Markovian dynamics for both $\Omega = 0$ (Fig. 4(a)) and $\Omega = 2\Gamma$ (Fig. 4(c)), the dynamical curves of the trace distance $D[\rho_1(t),\rho_2(t)]$ (state distinguishability) are very different regarding the points when it starts increasing, implying different mechanisms of information backflows in the two cases.

IV. MEMORY-KEEPING RESERVOIRS

In the above section, we have considered a qubit s interacting with two coupling modes m_1, m_2 which are dissipated respectively by two memoryless reservoirs $\mathcal{R}_1$, $\mathcal{R}_2$. Under these conditions, we have seen that the two modes are fully responsible for the memory effects of the overall environment on the qubit and their coupling can modify this effect. However, if the two modes are only components of the compound memory of the overall environment, the way their coupling changes the overall memory effects on the qubit is to be explored. We accomplish this analysis in this section. To this purpose, we consider a more complex situation where the coupling modes are dissipated by structured reservoirs $\mathcal{R}_1$, $\mathcal{R}_2$ exhibiting inherent memory effects and are therefore non-Markovian [1, 5].

We again assume the two modes and the corresponding reservoirs are initially in their ground states with only up to one excitation present in the total system. The qubit can initially be in a (normalized) superposition $|\psi\rangle_s = c_0(0)|0\rangle_s + h(0)|1\rangle_s$, so that the initial total state is $|\Phi(0)\rangle = |\psi\rangle_s|00\rangle_{m_1 m_2}|\overline{00}\rangle_{\mathcal{R}_1\mathcal{R}_2}$ with $|\overline{0}\rangle_{\mathcal{R}_n} \equiv \prod_k |0_k\rangle_{\mathcal{R}_n}$. The total evolved pure state then reads

$$\begin{aligned}
|\Phi(t)\rangle = {} & [c_0(0)\,|0\rangle_s + h(t)\,|1\rangle_s]\,|00\rangle_{m_1 m_2}\,|\overline{00}\rangle_{\mathcal{R}_1\mathcal{R}_2} \\
& + |0\rangle_s\,[c_1(t)\,|10\rangle_{m_1 m_2} + c_2(t)\,|01\rangle_{m_1 m_2}]\,|\overline{00}\rangle_{\mathcal{R}_1\mathcal{R}_2} \\
& + |0\rangle_s\,|00\rangle_{m_1 m_2} \sum_{n=1}^{2}\sum_k c_{n,k}(t)\,|\mathbf{1}_k\rangle_{\mathcal{R}_n}\,|\overline{0}\rangle_{\mathcal{R}_{\bar{n}}}, \quad (11)
\end{aligned}$$

where $|\mathbf{1}_k\rangle_{\mathcal{R}_n} \equiv |0\cdots 1_k\cdots 0\rangle_{\mathcal{R}_n}$ means that there is one excitation in the kth mode of the reservoir $\mathcal{R}_n$ and $\bar{n}$ is the complementary of n (i.e., $\bar{n} = 2$ if $n = 1$ and viceversa). The initial conditions of the coefficients appearing in $|\Phi(t)\rangle$ are $c_1(0) = c_2(0) = c_{n,k}(0) = 0$. From the Schrödinger equation [1], the time evolution of the total system in the interaction picture with the Hamiltonian of Eq. (2) is determined by the following differential equations

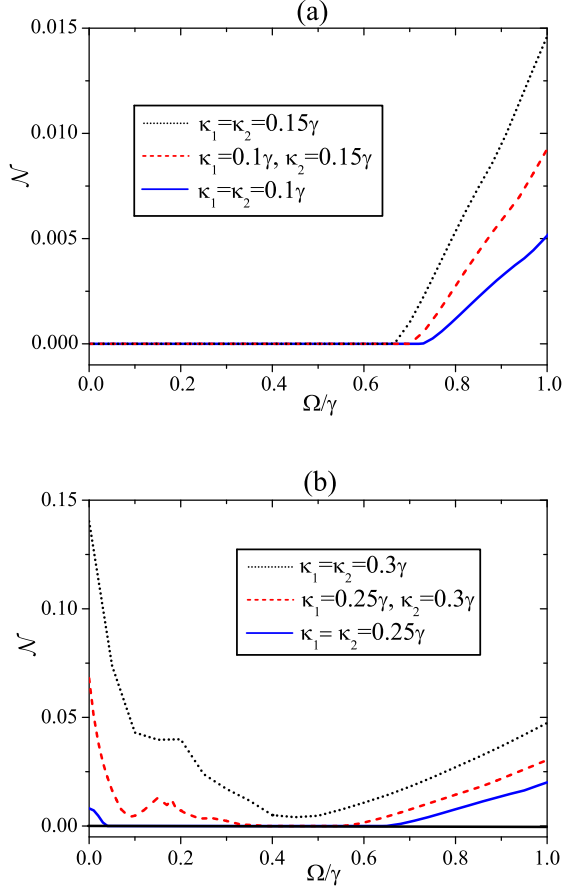$$\dot{h}(t) = -i\kappa_1 c_1(t) - i\kappa_2 c_2(t),$$

$$-\int_0^t \sum_k |g_{2,k}|^2 e^{-i\Delta_{2,k}(t-t')} c_2(t')dt'. \quad (13)$$

The sum $\sum_k |g_{n,k}|^2 e^{i(\omega_0-\omega_{n,k})(t-t')}$ in the above equations is recognized as the correlation function $f_n(t-t')$ of the reservoir $\mathcal{R}_n$, which in the limit of a large number of modes can be changed into an integration in terms of the spectral density $J_n(\omega)$ as [1]

$$f_n(t-t') = \int d\omega J_n(\omega) \exp[i(\omega_0-\omega)(t-t')]. \quad (14)$$

We take each reservoir $\mathcal{R}_n$ with a Lorentzian spectral density $J_n(\omega) = \gamma_n \lambda_n^2 / \{2\pi[(\omega-\omega_0)^2 + \lambda_n^2]\}$, where γ_n is the mode-reservoir coupling strength and λ_n^{-1} the reservoir correlation time [1, 5]. The two-point correlation function of Eq. (14) can be then expressed as $f_n(\tau) = \frac{1}{2}\gamma_n \lambda_n \exp(-\lambda_n|\tau|)$. Therefore, the amplitudes $h(t)$, $c_1(t)$ and $c_2(t)$ can be obtained by solving the first one of Eqs. (12) together with Eq. (13) by using the standard Laplace transform technique. The reduced dynamics of the qubit and the of other parts of the overall system are then determined by tracing out the opportune degrees of freedom from the evolved total state $|\Phi(t)\rangle$ of Eq. (11).

In Fig. 5, we plot the non-Markovianity $\mathcal{N}$ as a function of the scaled mode-mode coupling strength Ω/γ for different values of system-mode couplings κ_1, κ_2 and assuming $\gamma_1=\gamma_2=\gamma$. As shown in Fig. 5(a), when the memory effects of the two reservoirs alone (that is with $\Omega/\gamma = 0$) are not sufficient to make the system experience non-Markovian dynamics, the introduction of mode-mode coupling with sufficient strength can drive the Markovian dynamics to the non-Markovian one. The non-Markovianity is moreover proportional to the coupling strength for given values of κ_n and λ_n. This implies that the coupling of the modes, as constituents of the compound memory of the overall environment, can further enhance the memory effects of the latter on the qubit. On the other hand, if the system already undergoes non-Markovian dynamics without mode-mode coupling, the relation between the non-Markovianity and the mode-mode coupling results to be nonmonotonic. The mode-mode coupling reduces the non-Markovianity and can even transform a non-Markovian dynamics ($\mathcal{N} > 0$) into a Markovian one ($\mathcal{N} = 0$). Nevertheless, the further increase of the coupling strength Ω can recover and increase the non-Markovianity, as shown in Fig. 5(b). These behaviors are analogous to the ones found before for the case of memoryless reservoirs, a slight difference being that here the non-Markovianity exhibits oscillations as Ω/γ increases before reaching its minimal value (see Figs. 2(b) and 5(b)).

V. CONCLUSION

In this paper we have addressed the study of the effects of the coupling between two parts of a multiple environ-

FIG. 5: The non-Markovianity $\mathcal{N}$ as a function of the coupling constant Ω/γ between two modes that are dissipated by Lorentzian reservoirs with $\gamma_1 = \gamma_2 = \gamma$. (a) and (b) demonstrate weak and strong couplings between the qubit and the modes in terms of κ_1 and κ_2. The other parameters are chosen as $\lambda_1 = \lambda_2 = 0.5\gamma$ in (a) and $\lambda_1 = \lambda_2 = 0.8\gamma$ in (b).

$$\begin{aligned}
\dot{c}_1(t) &= -i\kappa_1 h(t) - i\Omega c_2(t) - ig_{1,k}^* e^{i\Delta_{1,k}t} c_{1,k}(t), \\
\dot{c}_2(t) &= -i\kappa_2 h(t) - i\Omega c_1(t) - ig_{2,k}^* e^{i\Delta_{2,k}t} c_{2,k}(t), \\
\dot{c}_{1,k}(t) &= -ig_{1,k}^* e^{i\Delta_{1,k}t} c_1(t), \\
\dot{c}_{2,k}(t) &= -ig_{2,k}^* e^{i\Delta_{n,k}t} c_2(t).
\end{aligned} \quad (12)$$

Integrating the last two equations with the initial condition $c_{n,k}(0) = 0$ ($n = 1, 2$) and inserting their solutions into the second and third equation above, one obtains two integro-differential equations for the amplitudes $c_1(t)$ and $c_2(t)$

$$\begin{aligned}
\dot{c}_1(t) &= -i\kappa_1 h(t) - i\Omega c_2(t) \\
&\quad - \int_0^t \sum_k |g_{1,k}|^2 e^{-i\Delta_{1,k}(t-t')} c_1(t')dt', \\
\dot{c}_2(t) &= -i\kappa_2 h(t) - i\Omega c_1(t)
\end{aligned}$$

ment on the dynamics of a quantum system. In particular, we have considered a qubit (the quantum memory) simultaneously interacting with two coupled bosonic modes (the control devices) which are in turns dissipated into memoryless or memory-keeping reservoirs. In the case of memoryless reservoirs, we have proven that the two cavity modes play the role of unique memory sources of the overall environment for the qubit and their coupling can be thus viewed as a coupling between two quantum memory sources. In the case of memory-keeping reservoirs, the two modes are instead the constituent parts of the total memory source of the overall environment and their coupling can be now meant as a coupling between two partial memory sources for the qubit. We have shown that in both cases the Markovian dynamics of the qubit, existing without the mode-mode coupling (the control parameter), can become non-Markovian by adjusting the control parameter over a certain threshold. Moreover, higher values of the control parameter enable larger non-Markovianity for the qubit. Differently, when the qubit evolution is already in a non-Markovian regime for a zero mode-mode coupling, a nonmonotonic relationship arises between non-Markovianity and control parameter. Namely, multiple crossovers from non-Markovian to Markovian regimes may occur by increasing the mode-mode coupling. This may appear surprising since, on the basis that increasing the coupling between memory sources for the qubit entails a transition from Markovian to non-Markovian regimes for the qubit, one expects that an increasing of the control parameter always induces an enhancement of memory effects on the qubit dynamics.

We remark that the behaviors above happen independently of the nature of the reservoirs. Our findings evidence that when the environment is composite the underlying physical mechanisms may be counterintuitive. The environmental coupling thus reveals as a powerful and effective tool to activate and harness quantum non-Markovianity of open systems. It is worth to notice that our system has the advantage to make it emerge in a clear way the effects of this coupling on the dynamics of a quantum system and, at the same time, to be simple enough to find feasibility within current experimental technologies, for instance in circuit QED [53] or in simulating all-optical setups [54]. Since non-Markovianity is linked to a dynamical recovery of the quantum coherence of a qubit [1, 6], our work highlights that engineering and exploiting suitably structured compound environments can supply useful developments for controlling and preserving quantum memory resources. It also motivates further studies regarding the effects of multiple environments on the dynamics of correlations in many-qubit systems.

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